Engineering Statics
Homework 6 Notes

The centroid of an area is defined as the distance of an area from an axis multiplied by that area, divided by the total area. You may think of it as a summary: if the entire area (or mass) could be concentrated at one location, how far from the given axis would the area (mass) have to be to be equivalent? Thus, it acts like a moment arm.

1. The centroid in the *x* direction would be defined as

$\bar{x} = \dfrac{\int x dA}{\int dA}$, keeping in mind that *dA* is *(h-y)dx* because the curve defined by y is the lower border of the area. Thus, the integral becomes $\bar{x} = \dfrac{\int x(h-y)dx}{\int (h-y)dx}$. The integral is probably best (easiest) done by parts, and treating the nominator and denominator separately. At this point, just substitute, multiply through with x, and integrate.

The centroid in the *y* direction can be done at least two ways. The way that doesn't require transforming the curve function (ack!) uses the general equation $\bar{y} = \dfrac{\int \frac{1}{2} y dA}{\int dA}$. This is because we are measuring the centroid in the *y* direction to be ½ the length of our sliver of area. But because our problem has the offset, the centroid of the sliver is *½ (h+y)* . We substitute *dA* as before to get $\bar{y} = \dfrac{\int \frac{1}{2}(h+y)(h-y)dx}{\int (h-y)dx}$. You may wish to simplify some part of this algebraically. As before, substitute thoug and integrate by parts. You can do yourself a favor by re-using the area.

2. Simlar to the first problem but easier because you don't have to subtract, I would be inclined to calculate the areas and centroids of each area, and then sum them where (Centroid_1 + Centroid_2)/(Area_1 + Area_2) which gets you the centroid of the entire shaded area.

3. For standard shapes, I like to set up a table:

i	$\bar{x}_i$	A_i	$\bar{x}_i A_i$
A			
B			
C			
Σ		$=\Sigma A_i$	$=\Sigma \bar{x}_i A_i$

The centroid can then be calculated using division $\bar{x} = \dfrac{\sum \bar{x}_i A_i}{\sum A_i}$.

This particular problem is made more complex by the difference in densities. Given that we are provided densities, we can assume unit lengths and I would modify the table to use mass rather than simply area.

i	$\bar{x}_i$	A_i	ρ_i	$A_i \rho_i$ $= unit\ mass_i$	$\bar{x}_i A_i \rho_i$
A					
B					
C					
Σ				$=\Sigma A_i \rho_i$	$=\Sigma \bar{x}_i A_i \rho_i$

The centroid can then be calculated using division $\bar{x} = \dfrac{\sum \bar{x}_i A_i \rho_i}{\sum A_i \rho_i}$.

4. This problem is best solved using the fact that it is symmetrical. Locate the geometric center.

5. The C-channels are identical, so I would be inclined to calculate the centroid for the C-channel lying on the x-axis for both the x and y centroids. With those values, I would set up a table similar to the one in the previous problem:

i	$\bar{x}_i$	A_i	$\bar{x}_i A_i$	$\bar{y}_i$	$\bar{y}_i A_i$
1					
2					
Σ		$=\Sigma A_i$	$=\Sigma \bar{x}_i A_i$		$=\Sigma \bar{y}_i A_i$

Remember to rotate and offset the C-channel that is standing. $\bar{x} = \dfrac{\sum \bar{x}_i A_i}{\sum A_i}$ and $\bar{y} = \dfrac{\sum \bar{y}_i A_i}{\sum A_i}$.

6. This problem is acually simpler (is this a word?) than the previous. Use the table format:

i	$\bar{x}_i$	A_i	$\bar{x}_i A_i$	$\bar{y}_i$	$\bar{y}_i A_i$
1					
2					
3					
Σ		$=\Sigma A_i$	$=\Sigma\, \bar{x}_i A_i$		$=\Sigma\, \bar{y}_i A_i$

The theory of Pappus (sometimes refered to as Pappus/Guldinus) holds that volume and surface area can be determined by calculating the centroid of the cross-section, and using that as a "sweeping arm" for either the cross-sectional area (for the volume), or the perimeter (for the surface area). Thus, to calculate a volume can be as simple as multiplying the cross-sectional area times the distance to the centroid (from the axis of the sweep) times the sweep in radians.

7. See above. Weight is determined by volume times density (but you knew that).

8. There are several ways to calculate the cross-sectional area and the centroid from the axis of rotation. The easiest way might be to chop the form into three cylinders. The cylinder on the left would have a cross-sectional area of 1000(=10x100)mm^2 and centroidal distance of 55mm. The middle cylinder would have a cross sectional area of 900mm^2 and a centroidal distance of 50mm, and the right cylinder would have a cross-sectional area of 1500(=150x10)mm^2 and a centroid distance of 25mm.
For this problem you can calculate the volumes individually and then summing them, or calculating the centroid for the three sections, and then dividing by the total cross-sectional area. This would give you the centroid of the entire piece. Then multiply by the cross-sectional area, and the length of the sweep in radians (which would be 2π).