Engineering Dynamics
Homework 7 Notes

About Energy and Rotation:
One of the things to consider is that an object can have kinetic energy in two forms: rotational, and linear. The linear form of kinetic energy is commonly from the center of mass. The rotational form of kinetic energy is also commonly from the center of mass.

But if you attempt to describe the kinetic energy of an object using both linear and rotational forms, you must take them from the same location.

For example, a lawn mower blade spins about a center, and moves along horizontally. The total energy can be described by the moment of inertia about the center of mass, and the linear motion of the center of mass. No problem.

But if the same lawn mower blade is rotated about one of the ends, or for convenience sake, the moment of inertia is calculated from the end using the parallel axis theorem, and the end *does not have linear motion*, then there is no need to calculate the linear component of the energy. It has already been compensated for when the moment of inertia was redefined.

1.
Note that they mention a force at a distance from the axis applied over an angle, in addition to a resistive moment. This is analogous to force times distance, or work. The resisting moment is friction. *My* preference is to put friction on the left side of the equation using subtraction, but that is up to you.

$$Pr\theta - M_f\theta = \tfrac{1}{2}I\omega^2$$

where r is the distance to the applied force, P. The moment of inertia, I, can be calculated using the equation/formula in the inside cover of the text and applying the parallel axis theorem.

$$I = 1/12\ mb^2 + mr^2$$

where r is the distance from the center of mass to the axis of rotation and b is the length of the plate perpendicular to the axis of rotation. Remember that there are 4 doors. Solve for ω.

2.
The wheel has an initial energy, $\tfrac{1}{2}I\omega^2$. The moment of inertia for the disk is $I = mk^2$. We subtract the frictional moment until it stops (angular velocity is zero). The frictional moment is defined by a frictional force, F_f, times the distance, r. The frictional force is the reaction force at B times the coefficient of friction, μ.

$$\tfrac{1}{2}I\omega^2 - M_f\theta = 0$$

Solve for θ. The reaction forces at *A* is a Statics problem and can be determined using a free body diagram. Don't forget to convert the angular solution to revolutions.

3.
The system has an initial energy, which is the two rods, *AB* and *CD,* with kinetic energy due to angular motion, but also kinetic energy due to linear motion (measured from the center of mass); as well as the kinetic energy of the bar, *AC*. In addition, there is an applied moment which is applied over 90º in addition to a force applied over a distance equal to the length of the link. This results in a total kinetic energy as well as a change in potential energy. Note that the changes in potential energy for the links, *AB* and *CD,* is measured from their centers of mass. The same is true for the bar *AD*.

Using what was mentioned before, if you calculate the moment of inertia at the ends, then you can ignore having to calculate the kinetic energy due to the linear movement of the center of mass.

This is a very long equation with lots of terms, but it all makes sense when you put all of the pieces together.

4.
This problem needs to broken into two distinct segments. One where the diver is in an initial energy state and then curls forward until his center of mass is even with the top of the diving platform. At this point he has converted a change in potential energy into rotational and linear energies, with a relationship between those two.

The second segment requires us to determine how long it took for the diver to fall 40 feet. If we know the time, we can apply this to the angular velocity determined from the first segment to calculate the number of rotations before impacting the water.

$I = mk^2$ *notice that we have not taken the moment of inertia from point A*

$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$ *for the first segment where* $v = \omega r$

In this part, we will call the velocity v_i because we will need it in the next segment. For the second segment, the time to fall can be calculated using projectile motion, or

$$s = v_i t + \frac{1}{2}gt^2$$

where $\theta = \omega t$ which is analogous to $x = vt$. We must solve for the time, *t*. Convert the angle θ to revolutions. The angular velocity remained constant through the fall because nothing acted to change it.

5.

This is an energy problem, where the force P is applied over a given distance. This results in a change in kinetic and potential energy for the system. The pulley has both rotational and linear energy components. The crate has only linear kinetic energy, but note that the velocity of the crate is twice that of the pulley. The potential energy also changed so that while the pulley moved up so far, the crate moved up twice as far.

I would put the incoming work, the force P times the applied distance, on the left side of the equation, and the resulting kinetic and potential energies on the right side.

6.
Again, an energy equation with changes in kinetic and potential energy. The change in potential energy is the component driving this situation. I would probably put the potential energy on the left side of the equation, with the resulting kinetic energies on the right side of the equation. The relationship between the angular velocity of the spool and the velocity of the man would be $v = \omega r$.

7.
A classic problem with a twist. We know that the girl will be rotating, so we have to consider that. If the moment of inertia for the girl is calculated from the top of the swing, we would get

$$I = mk^2 + mr^2 \qquad \textit{parallel axis theorem}$$

The energy equation would look like this because we are measuring the rotational energy from a point that has no linear motion:

$$mgh = \tfrac{1}{2}I\omega^2$$

Solve for ω.

If we attempted to solve this using linear kinetic energy, we would get the same solution, but with more work on our part.

$$I = mk^2$$
$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

Resolve the relationship between v and ω where $v = \omega r$. Solve for ω.

8.
The window's moment of inertia about point *A* is

$$I = mk_A^2 \qquad \textit{Note that the radius of gyration is about point A, not G}$$

There is potential energy initially, measured by the change in the height of point *G*, and no spring energy. In the end we only have kinetic energy of the window and spring energy. The hardest part is figuring out what the spring energy is.

$$mgh = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}kx^2$$

The distance *h* is the change in the height of the center of mass, *G*. Given that the spring is initially unstretched, the stretched distance is the length of the cord from *B* to *C* when the window is all the way down. The velocity at *B* can be determined from the angular velocity.

9.
The initial condition is the change of the potential energy of the slender rod. At angle θ, the rod has an angular kinetic energy, the block has a linear kinetic energy as well as a change in potential energy. The relationship between the angular and linear velocities of the slender rod is $v = \omega r$. The hardest part of this problem is calculating how much the distance between points *B* and *C* changes because that is the distance that the block, *D*, moved up. A secondary challenge is determining the velocity of the cord attached to block *D*. The velocity at point *B* needs to broken into a vector. I would recommend drawing a diagram of the vector components at point *B* in order to get the correct angles. Additionally, I would say that the velocity of the cord is going to be less than the velocity of point *B*.