Engineering Dynamics
Homework 6 Notes

1.
This one looks either ugly or too simple. And it is neither. First, consider that an awful lot of information is provided, so you can assume it will be used. It will be. The only horizontal force is that caused by the rear tire in contact with the road, and that is derived from the force down on the rear axle, or $R_A\mu$. (I'm going to call the horizontal force F_A, and the vertical reaction force R_A., thus $F_A = R_A\mu$). The car is not accelerating vertically, so we can use statics to determine that $\Sigma F = 0 = G_Tg + G_bg + G_Eg + R_A + R_B$ (where g = $9.81 m/s^2$). I would be inclined to consider the sum of moments about the point of contact, A. Remembering that the car is accelerating forward, but not experiencing angular acceleration, thus

$$\Sigma M = 0 = -F_A(.55) - F_A(.4) - F_A(.6) + F_A(1.25) + F_A(1.25+1.46) + R_A(1.25 + 1.46)$$

Three equations, three unknowns. Q.E.D.

2.
This is rectilinear motion, which means that there is no rotation *per se* on the T shaped structure. We need to determine the acceleration at point E, which is the sum of tangential and normal accelerations. Tangential acceleration is αr. Normal acceleration is v^2 / r . Remember that these are at 30º angles. These tell us what the acceleration at point E actually is.

The sum of forces is mass times acceleration, or $\Sigma F = ma$. Mass is given (don't forget to convert to slugs!), and we just determined acceleration. (Remember that both force and acceleration are vectors, which should make it easier to calculate.)

The moment is force times distance, or mass times acceleration time distance. Calculate this for each component about point E and sum them.

3.
The forces on the box can be described by a force on the bottom going up, a horizontal force along the bottom going to the right, and the weight (going down). This will be equal to the mass time the resulting acceleration, or $\Sigma F = ma$. The acceleration, a, would be αr, where r is the length of the links.

The acceleration in the x direction, a_x, would be described as $\alpha r \cos(30^o)$, and the acceleration in the y direction would be described as $\alpha r \sin(30^o)$. Putting these components into the form $F=ma$, we would get

$$\Sigma Fx = ma_x = m\alpha r \cos(30^o) = F_N\mu$$
$$\Sigma Fx = ma_y = m\alpha r \sin(30^o) = F_N - mg$$

With two equations and two unknowns, this should be solvable.

4.
A classic problem, we would solve for α by summing the moments about point A. The normal force is determined by the angular velocity, thus

$A_x = m\omega^2 r$ where r is the distance to the center of mass.
$A_y = m\alpha r - mg$

But we are unable to solve for a yet, so we need another relationship.

$$\Sigma M = I\alpha = A_y\, r$$

The moment of inertia about the centroid is $1/12\, ml^2$. Solve for α.

The horizontal force at A is the normal acceleration times the mass, or $m\omega^2\, \ell/2$. The vertical force is the acceleration caused by the angular acceleration minus the acceleration due to gravity, or $m\alpha\ell/2\text{-}mg$.

5.
We will call the tension in the cord T. Summing the moments about the center we get

$$T\,r = I\,\alpha \qquad where\;\; I = mk^2$$

The value r is the smaller radius. Summing the forces for the mass we see

$$\Sigma F = ma = T - mg$$
$$a = \alpha\, r$$

Solve for T, substitute into the first equation, and then solve for α. Knowing α, we can go after the angular velocity. The total distance would be described as $s = \theta r$. Given r and s, we can calculate the angular rotation in radians, θ. We then plug this into our known kinematics relationship

$$\omega^2 = \omega_o^2 + 2\alpha(\theta\text{-}\theta_o)$$

Solve for ω.

Personally, I would look at this using energy, where

$$mgh = ½\, mv^2 + ½\, I\omega^2$$

The relationship between v and ω is $v = \omega r$. The moment of inertia, I, is mk^2.
Solve for ω. (Don’t forget to convert pounds to slugs.)

6.
I would look at summing the moments because there is direct linkage between disk A and B.

$$\Sigma M = M_{APPLIED} = I_A\alpha_A + I_B\alpha_B$$

Where I_{DISK} is 3/2 mr^2

The acceleration at the contact point between disk A and disk B is the same, thus we can say

$\alpha_A r_A = a = \alpha_B r_B$, and thus $\alpha_B = \alpha_A\, r_A\, /\, r_B$

Substitute and solve.

7.
We would solve for α by summing the moments about point A. The hardest part of this problem is just figuring out the moment of inertia about point O, and the distance from O to the center of mass. (Let's call that r.)

$3/2\ (8) + (3 + .75/2)(13.5) = r\ (8 + 13.5)$ Solve for r.

$I = 1/12\ (8)\ (3/2)^2 + [\ 3/2(13.5)(.75)^2 + (13.5)\ (3+.75/2)^2\ ]$

Note that we had to use the parallel axis theorem to adjust the moment of inertia for the disk.

$\Sigma M = mgr = I\alpha + m\alpha r^2$ Solve for α.

8.
Summing the moments about point B, we get

$$\Sigma M = Fr = I_B\alpha$$

$F(4+1.5+.4) = (I_G + mr^2)\alpha$
$F(4+1.5+.4) = (I_G + m(4+1.5)^2)\alpha$
$I_G = mk^2$

Notice that is applying the parallel axis theorem where r is the distance from the point of rotation to the center of mass. Substitute and solve for α.

Because there is no angular velocity, there is no normal acceleration.

9.
Let's call the point of contact between the tire and the incline C. Summing the moments about point C, we can simplify at least part of the problem because we can ignore F_N and F_f for now.

$$\Sigma M_C = mg(2-.75)\sin 20^o = I_C\alpha$$

where $I_C\alpha = (I_G + mr^2)\alpha$ (notice the application of the parallel axis theorem).

Looking at the normal force, we know that the sum of the forces must be equal to zero otherwise the tire would be accelerating into space (or worse).

$$\Sigma F_N = 0 = -mg\cos 20^o + F_N + m\omega^2 r$$

The tangential force or frictional force can be described by

$$\Sigma F_t = -mg\sin 20^o + F_f = m(1.25)\alpha$$

10.
No gravity in this problem (everything lies in the horizontal plane). Because of the geometry, all the end points have the same linear acceleration, a_C., pointing downward Starting at point A, I am just looking at link AB. The moment is

$$\Sigma M = M = I_{AB}\alpha_{AB} + F_B r_{AB} + I_{BC}\alpha_{BC}$$

The moment caused by the connection to link BC is $F_B r_{AB}$. Summing the forces to determine F_C, we could write

$$\Sigma F = \Sigma ma \qquad \text{or} \qquad F_B = m_{BC}a_C + m_C a_C$$

The applied moment also has to cause angular acceleration to link BC. The angular acceleration of BC is a little trickier in that it is derived from the acceleration of point B. The acceleration of point B, a_B, is $\alpha_{AB}\, r_{AB}$. We have to use a component perpendicular to link BC. To determine α_{BC}. We use the component 3/5 a_B. (The given geometry is a 3-4-5 triangle. An exercise in sketching to get the correct ratio.) We can then determine α_{BC} from the center of mass of link BC.