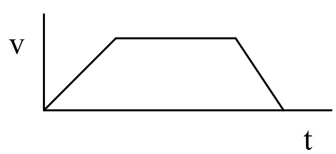
Engineering Dynamics
Homework Notes 1

1.
The distance traveled as well as the velocity are given, and the acceleration is unknown, thus we would consider using $v^2 = v_o^2 + 2a(x - x_o)$. Solve for acceleration, *a*. To solve for the time you may want to use $x = \frac{1}{2}at^2$, solving for time, *t*.

2.
You are given the expression for the displacement. It should be pretty straight forward to get the first value wanted. The velocity can be determined by taking the derivative with respect to time, *t*. You need to solve for values of *t* where the velocity is zero. Then calculate the displacement between each span to get the total distance traveled.

3.
Draw a *v-t* curve.



Acceleration is the slope of the curve (line). Knowing that acceleration is the change of velocity (the vertical component) divided by the change in time (the horizontal component). The last segment goes from a velocity of 6 f/s to 0 over a period of 0.6 seconds.

Displacement is the area under the curve, so you need to calculate the area under each section and sum them up for the distance traveled. The average velocity is the total distance divided by the time.

4.
You are given an expression for the acceleration, and the displacement in an attempt to get the velocity. The first thought is to use the equation $v^2 = v_o^2 + 2a(x - x_o)$, but this would only work if the acceleration is constant, but it isn't. So, you need to use original version of that equation, so that you get something like:
$v^2 = v_o^2 + 2\int(6 + 0.01s)\, ds$
solving for *v*. The time, *t*, has to be determined using a similar expression:
$t = \int_{so}^{s} \frac{ds}{v^2}$ (where s is the displacement)

5.
This one shouldn't be so bad, because acceleration is the slope of the curve segment. The distance is the area under each curve segment. The average velocity is the total distance divided by the total time.

6.
You are given the position in terms of time, t. The velocity is the first derivative of the position vector. The particle's velocity requires that you calculate the velocity for each vector component, and then use the Pythagorean Theorem. Ditto for acceleration.

7.
Acceleration is the derivative of velocity. Like the previous problem use the Pythagorean Theorem to get the acceleration, a. Using each component, calculate the angle by using the following process:
Calculate the magnitude of the acceleration (this is the hypotenuse for our triangle). (a)
$a_x = a \cos \alpha$
Solve for the angle, α.

Do the same for the other angles.

8.
In most ways this problem is pretty straight forward. It's just that the derivative might confuse you. To get the velocity and acceleration you just have to take derivatives and then solve for t. But its easy to forget that θ is a function t. Everyone who thought that the derivative of $\sin^2\theta$ was $2\cos\theta\sin\theta$ might want to take note. Because θ is a function of t, we have to take the derivative of θ also. Thus, the ***i*** component in velocity vector ***v*** is
$2\dot{\theta}\cos\theta\sin\theta$
(Those of you unfamiliar with the notation, $\dot{\theta}$ is the derivative of θ with respect to time. Similarly, $\ddot{\theta}$ is the derivative of $\dot{\theta}$ with respect to time.)
The ***i*** component in acceleration vector ***a*** is
$\dot{\theta}^2(2\cos^2\theta - \sin^2\theta) + 2\ddot{\theta}\cos\theta\sin\theta$

By itself, this doesn't do much, but if you take the derivatives of θ, you can substitute, then substitute for t, and solve numerically.
$\theta = 3t^2$
$\dot{\theta} = 6t$
$\ddot{\theta} = 6$

Alternatively, if you have a calculator similar to the TI-89 you can enter the entire expression in terms of time, t, and take the derivatives, substituting for t, and solve.

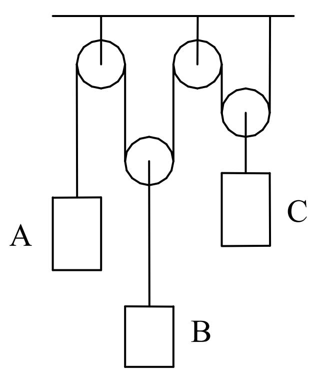
9.
Calculate the displacement from the starting point, producing x and y. (x=108 * 4/5, y=108 * 3/5 + 6) Because projectile motion is being used, we can write equation of motion:
$x = v_A \cos \theta_A t$
$y = v_A \sin \theta_A t - \frac{1}{2} g t^2$
Two equations, two unknowns.

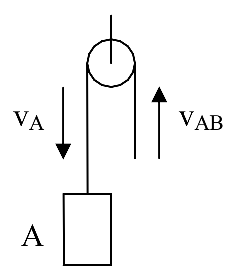
10.
$a_n = v^2/r$
The tangential acceleration is the change in velocity divided by the change in time
a_t=(250-160)/4
The magnitude of the acceleration is easily determined using the Pythagorean Theorem.
$a^2 = a_n^2 + a_t^2$

11.
Take the derivative of the function twice:
$\theta = 5t^2$
$\dot{\theta} = 10t$
$\ddot{\theta} = 10$
The acceleration components can be determined where $a_n = \dot{\theta}^2 r$ and $a_t = \alpha r = \ddot{\theta} r$. Like the previous problems use the Pythagorean Theorem to determine the magnitude of the acceleration, a.

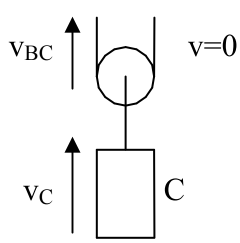
12.
The velocity can be determined using $v = \omega r = \dot{\theta} r$. Acceleration is a bit trickier where $\overline{a} = (\ddot{r} - r\dot{\theta}^2)\overline{u}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\overline{u}_\theta$, but it gets easier because $\ddot{r} = 0$ and $\dot{r} = 0$. Solve for the components and use the Pythagorean Theorem.

13.
Just a little diagram with some labels (it makes it easier to talk about these):

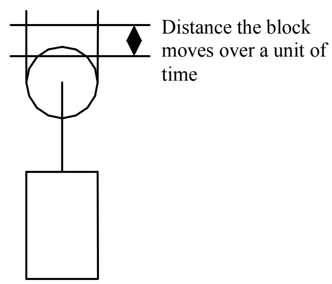




Looking first at block A, we can see that the velocity of the cable (we're calling AB) is the same as the velocity of block A because the supporting pulley is stationary.

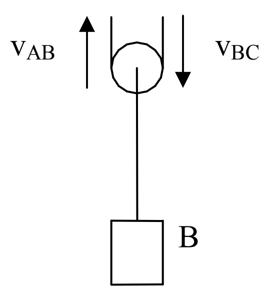


Looking next at block C, the other side, we have a more interesting problem. The velocity of cable BC (the cable between the pulleys supporting blocks B and C) will be twice that of the block. Why?



Notice that for a given unit of time, the block moves some distance. But you can see that twice as much cable is displaced. If one end of the cable is stationary, then the cable on the other side must be moving at twice the velocity of the block. A general way a saying this is that the average of the velocities of the two cables is the velocity of the block.

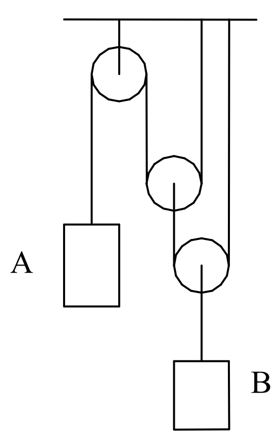
$v_{BLOCK}=(v_{CABLE1} + v_{CABLE2}) / 2$



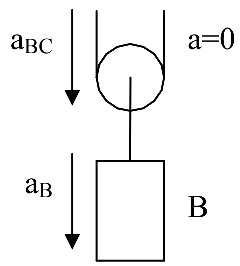
This generalization works well for us here because we not only have two cables related to block B, but both are moving – and in opposite directions(!). The good news is that we can average the two velocities to determine the velocity of block B. Be sure to select a positive direction when using velocities (is *up* positive, or is *down*?).

14.

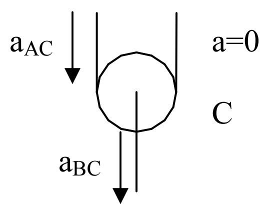
Just a little diagram with some labels (it makes it easier to talk about these):



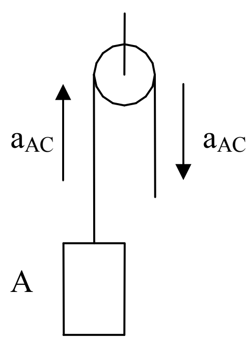
To make things a little easier, I'm going to call the pulley in the center C.



Knowing the acceleration of block B is helpful because we can determine the acceleration of the cable between block B and pulley C.



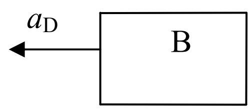
Given the acceleration of the cable connecting block B and pulley C, we can use the fact that like velocity (from the previous problem), the acceleration of the cable connecting block A and pulley C is a function of the average, i.e. $a_{BC} = (a_{AC}+0)/2$

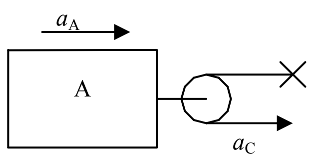


We can see that the acceleration for block A will be the same as the acceleration for the cable connecting block A and pulley C.
Like more conventional calculations, velocities can be derived from acceleration by multiplying by time, t, $v=at$. Distance can be determined using $s= \frac{1}{2} at^2$.

15.

Just a little few diagrams with some labels (it makes it easier to talk about these):





From the previous problems we can see that the acceleration of block A will be half of the cable pulling it, so $a_A = ½\, a_C$. The acceleration at B is pretty obvious. The tricky part of this problem is that the acceleration a_C is not a constant – it's a function of time, t. Thus, we have to integrate to get the expressions for velocity and displacement, s.

$a_C = 8t^2$ $\quad a_A = 4\,t^2$

$v_C = 8/3\,t^3$ $\quad v_A = v_{Ao} + 4/3\,t^3$

$s_C = 2/3\,t^4$ $\quad s_A = s_{Ao} + v_{Ao}t + 1/3\,t^4$

We can write the equations of motion for block B:

$v_B = v_{Bo} + ½\, a_Bt^2$

$s_B = s_{Bo} + v_{Bo}t + ½\, a_Bt^2$ (note that a_B is a negative value because it is to the left)

We know that both blocks start at rest, so $v_{Ao}=0$ and $v_{Bo}=0$.

We need to solve for the time when the two displacements are the same. If we place an arbitrary 'zero' at the left edge of block A (fixed to a non-moving reference frame), then we could say that $s_{Ao}=0$ and $s_{Bo}=1$, and we are looking for the time when $s_A=s_B$. Thus, our expressions may look like this:

$s_A + s_B = 0$

$1/3\,t^4 - (1 + ½\,(-7)\,t^2\,)= 0$

Solve for t*. The relative velocity is commonly expressed as $v_{B/A}$. The common way to interpret this is v_B-v_A (or $v_{B/A}= v_B$-v_A). A way to think about this is to imagine the velocity of block B, which we already know to be negative, and made to seem even more negative by the positive velocity of block A.

* I looked at this solution using MathCAD and got four solutions, but only one was positive and not imaginary.